Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 17: Composite Functions

Functions can be added, subtracted, multiplied, and divided to produce other functions.

Definition 1. **Sum and Difference of Functions:** *The* ***sum*** *of two functions is defined by* $(f+g)(x) = f(x) + g(x)$*. The* ***difference*** *of two functions is defined by* $(f-g)(x) = f(x) - g(x)$*.*

Definition 2. **Product and Quotient of Functions:** *The* ***product*** *of two functions is defined by* $(fg)(x) = f(x).g(x)$*. The* ***quotient*** *of two functions is defined by* $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$*.*

Example 1. *Let* $f(x) = x^2 - 6x + 8$ *and* $g(x) = x - 2$*. Then*

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x^2 - 6x + 8 + (x-2)\\ &= x^2 - 6x + 8 + x - 2\\ &= x^2 - 5x + 6.\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= x^2 - 6x + 8 - (x-2)\\ &= x^2 - 6x + 8 - x + 2\\ &= x^2 - 7x + 10.\end{aligned}$$

$$\begin{aligned}(fg)(x) &= f(x).g(x)\\ &= (x^2 - 6x + 8)(x-2)\\ &= x^3 - 8x^2 + 20x - 16.\end{aligned}$$

$$\begin{aligned}(\frac{f}{g})(x) &= \frac{f(x)}{g(x)}\\ &= \frac{x^2 - 6x + 8}{x-2}\\ &= \frac{(x-2)(x-4)}{x-2},\ x \neq 2\\ &= x - 4,\ x \neq 2.\end{aligned}$$

Definition 3. **Composition of Functions:** *Let* $f(x)$ *and* $g(x)$ *be two functions, then the composition of the function* f *with the function* g *is written as* $f \circ g$ *and is defined by the equation*

$$(f \circ g)(x) = f(g(x)),$$

where the domain of $f \circ g$ *consists of those values* x *in the domain of* g *for which* $g(x)$ *is in the domain of* f*.*

Example 2. *Let* $f(x) = x^2 - 6x + 8$ *and* $g(x) = x - 2$. *Then*

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x-2) \\ &= (x-2)^2 - 6(x-2) + 8 \\ &= x^2 - 2x + 4 - 6x + 12 + 8 \\ &= x^2 - 8x + 24.\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(x^2 - 6x + 8) \\ &= x^2 - 6x + 8 - 2 \\ &= x^2 - 6x + 6.\end{aligned}$$

Example 3. **<u>Finding the Domain of a Composite Function:</u>** *Let* $f(x) = x^2$ *and* $g(x) = \sqrt{x}$. *Then*

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x})$$

Since, the domain of $g(x) = \sqrt{x}$ *is* $[0, \infty)$, *then*

$$\begin{aligned}(f \circ g)(x) &= (\sqrt{x})^2,\ x \geq 0 \\ &= x,\ x \geq 0.\end{aligned}$$

Therefore, the domain of $(f \circ g)(x) = x$ *is* $[0, \infty)$.

Definition 4. **<u>Decomposition of a Function:</u>** *Sometimes it is useful to use the concept of composition to* ***decompose*** *a function into simpler functions. For example,* $H(x) = \frac{1}{\sqrt{2x^2+1}}$ *can be written as* $H(x) = f(g(x))$ *with* $f(x) = \frac{1}{\sqrt{x}}$ *and* $g(x) = 2x^2 + 1$.

Example 4. *Decompose* $H(x) = (x^2 - 3)^{10}$ *into* $(f \circ g)(x)$.

<u>Solution:</u> $H(x) = f(g(x))$ *with* $f(x) = x^{10}$ *and* $g(x) = x^2 - 3$.